EFFECT OF POLYMER ADDITIVES ON A THREE-DIMENSIONAL TURBULENT BOUNDARY LAYER

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By means of an integral method the characteristics of the three-dimensional boundary layer are calculated with allowance for the effect of polymer additives using the Meyer correlation.

1. Numerous theoretical and experimental studies of the effect of polymer additives on the flow characteristics of viscious fluids have now been carried out, but mainly for flow in pipes and channels or geometrically simple boundary layers (plate, plane profile or body of revolution, disk) [1-6]. As a rule, the known experimental results for three-dimensional boundary layers relate solely to the overall drag reduction effect, information on the structure of the layer and its distributed characteristics being lacking. In this connection, it is useful to carry out a theoretical analysis of the effect of polymer additives and the Reynolds number on the distributed characteristics of the three-dimensional boundary layer.

In the method employed for the boundary layer calculations [7, 8] the effect of polymer additives on the flow is taken into account by varying the velocity profile in accordance with the Meyer correlation [9] for the motion of a body in a weak homogeneous polymer solution.

In solving the problem of three-dimensional viscous flow past bodies considerable importance attaches to the choice of coordinate system and to the basic assumptions associated with the simplification of the system of equations of motion.

In this study we have used a triorthogonal curvilinear flow coordinate system in which the longitudinal coordinate X is reckoned along the streamlines located on the surface of the body in an ideal fluid, the transverse coordinate Y along the equipotential lines on the body surface, and the Z coordinate along the outward normal to the surface. In this coordinate system the geodesic curvatures of the streamlines and equipotential lines are given by the relations

$$K_{12}=\frac{1}{h_1h_2}\,\frac{\partial h_1}{\partial Y},\quad K_{21}=\frac{1}{h_1h_2}\,\frac{\partial h_2}{\partial X},$$

where $h_1 = u_\delta^{-1}$, $h_3 = 1$, u_δ is taken equal to the ratio of the inviscid potential flow velocity at the surface of the body to the free-stream velocity.

The use of a flow coordinate system orthogonal only at the surface of the body allows us to confine ourselves to thin boundary layer theory, in which the Lamé parameters do not depend on the Z coordinate and the characteristic thicknesses of the layer must be much smaller than the radii of curvature of the surface exposed to the flow.

In the coordinate system employed the differential equations of motion that determine the flow under the usual assumptions of thin boundary layer theory and the continuity equation take the form indicated in [8], and by integrating these equations across the boundary layer and going over to dimensioness quantities we can obtain relations which, after making the assumption that the cross flow is small ($v < u$, $\partial u/\partial y \ll \partial u/\partial x$ and $\partial v/\partial y \ll \partial v/\partial x$), we can write in the form [7, 8]:

$$\frac{1}{h_1}\,\frac{d\delta_{11}}{dx}+\frac{du_\delta}{dx}\,(2\delta_{11}+\delta_1)+k_{21}\delta_{11}=\frac{c_f}{2},$$
$$\frac{1}{h_1}\,\frac{d\delta_{21}}{dx}+2\delta_{21}\left(\frac{du_\delta}{dx}+k_{21}\right)-k_{12}(\delta_{11}+\delta_1)=\frac{c_f}{2}\,\mathrm{tg}\,\beta, \tag{1}$$

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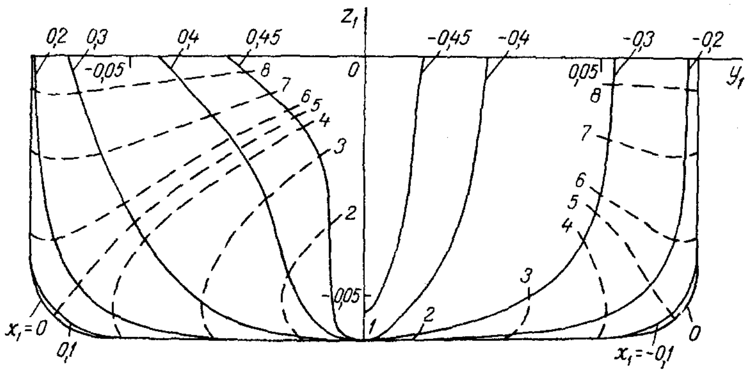

Fig. 1. Lower half of three-dimensional body (the numbers on the broken curves are the numbers of the streamlines).

$$\frac{1}{h_1}\frac{d}{dx}(\delta-\delta_1)+(\delta-\delta_1)\left(\frac{du_\delta}{dx}+k_{21}\right)=E(H_1),$$

where $E(H_1) = 0.0306\ (H_1 - 3)^{-0.653}$,

$$\delta_1=\int_0^\delta\left(1-\frac{u}{u_\delta}\right)dz,\quad \delta_2=-\int_0^\delta\frac{v}{u_\delta}dz,\quad \delta_{11}=\int_0^\delta\left(1-\frac{u}{u_\delta}\right)\frac{u}{u_\delta}dz,$$

$$\delta_{12}=\int_0^\delta\left(1-\frac{u}{u_\delta}\right)\frac{v}{u_\delta}dz,\quad \delta_{21}=-\int_0^\delta\frac{uv}{u_\delta^2}dz,\quad \delta_{22}=-\int_0^\delta\frac{v^2}{u_\delta^2}dz, \tag{2}$$

$$c_f=\frac{2\tau_{wx}}{\rho u_\delta^2},\quad c_f^*=\frac{2\tau_{wy}}{\rho u_\delta^2},\quad u=\frac{U}{U_0},\quad v=\frac{V}{U_0},$$
$$z=\frac{Z}{L},\quad y=\frac{Y}{L},\quad x=\frac{X}{L},\quad H_1=\frac{\delta-\delta_1}{\delta_{11}}, \tag{3}$$
$$k_{12}=K_{12}L,\quad k_{21}=K_{21}L.$$

In order to close system of equations (1) we must assign the mean velocity profiles in the longitudinal and transverse directions. The profile of the longitudinal velocity component may be taken [8] in the Coles form, and that of the transverse component in the Mager form:

$$\frac{U}{v^*}=\frac{1}{\varkappa}\ln\frac{Zv^*}{\nu}+\frac{\Pi}{\varkappa}\left(1-\cos\frac{\pi z}{\delta}\right)+B,$$
$$V=U\,\mathrm{tg}\,\beta\left(1-\frac{z}{\delta}\right)^2, \tag{4}$$

where $v^* = \sqrt{\tau_{wx}/\rho}$; $\Pi = \Pi(x)$; $\varkappa = 0.4$. In an ordinary viscous fluid [8] the quantity B is the second constant of turbulence ($B = B_0 = 5.2$). In our case, as distinct from [8], we can relate B to the characteristics of the polymer solution, for example by means of the relation [9]

$$B_0=\begin{cases}B_0, & v^*<v_0^*,\\ B_0+\alpha\lg\dfrac{v^*}{v_0^*}, & v^*\geqslant v_0^*,\end{cases} \tag{5}$$

in which α is a parameter that takes into account the properties of the polymer and its concentration in the solution, and v^*_0 corresponds to the onset of the drag reduction effect due to the introduction of the polymer into the flow.

Expressions (4) make it possible to obtain the necessary equations relating the characteristic thicknesses of the boundary layer. Using these, at the outer edge of the boundary layer we obtain the friction law

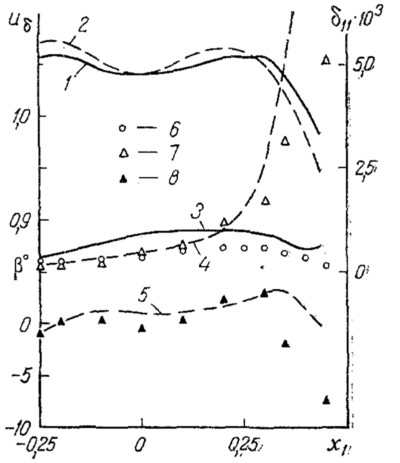

Fig. 2

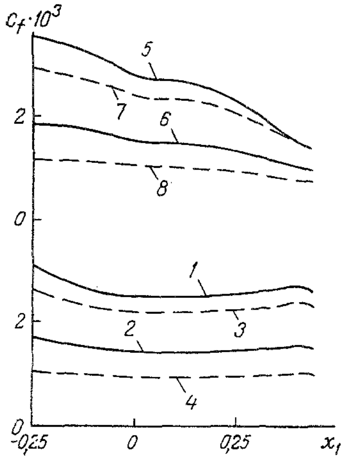

Fig. 3

Fig. 2. Distribution of velocity u_δ over the surface of a body in a potential flow, momentum thickness in longitudinal direction δ_{11}, and downwash angle β along length of body for streamlines 1 (continuous curves) and 5 (broken curves): 1, 2) u_δ; 3, 4, 6, 7) δ_{11}; 5, 8) β; 1-5) calculation; 6-8) experiment [10].

Fig. 3. Distribution of local coefficient of friction c_f along length of body for $\alpha = 0$ (continuous curves) and $\alpha = 10$ (broken curves): 1-4) for streamline 1; 5-8) for streamline 5; 1, 3, 5, 7) for Re = 10^7; 2, 4, 6, 8) for Re = 10^9.

$$\sqrt{\frac{2}{c_f}} = \frac{1}{\varkappa}\ln\left(\delta \operatorname{Re} u_\delta \sqrt{\frac{c_f}{2}}\right) + \frac{2\Pi}{\varkappa} + B, \tag{6}$$

where $\mathrm{Re} = U_0 L/\nu$.

The system composed of Eqs. (1) with substitution of the integral thicknesses (2) in accordance with (4) and Eq. (6) differentiated with respect to the longitudinal coordinate with allowance for (5) is a closed system of ordinary differential equations in δ, c_f, Π and β, whose integration for given initial conditions and velocity distribution at the outer edge of the boundary layer makes it possible to find all the integral characteristics.

2. In order to make calculations in accordance with this method, we compiled a program in ALGOL-60 for the BÉSM-6 computer. The calculations were carried out in conformity with the experimental conditions of [10] with Re = $5.2\cdot 10^6$ using the experimentally determined initial data at $x_1 = -0.3$ for an elongated body with two planes of symmetry and cross sections (x_1 = const) whose shape is represented by the continuous curves in Fig. 1. The Cartesian coordinate system x_1, y_1, z_1 moving with the body has been made dimensionless by dividing by the body length L; the origin is located on the line of intersection of the planes of symmetry in the middle of the body; the positive direction for readings along the x_1 axis is from bow to stern. In the same figure the broken curves represent a series of streamlines on the surface of the body for inviscid flow. The velocity and pressure distribution along these lines was taken from [10].

In order to estimate the reliability of the calculations, the results were compared with the experimental data (see Fig. 2). Over most of the body length the agreement is satisfactory. As a result of the various limitations of the method employed, at the stern of the body only qualitative agreement between calculation and experiment is observed.

This comparison makes it possible to use the method proposed to analyze the effect of the Reynolds number and the introduction of small amounts of high-molecular-weight polymers into the flow on the characteristics of the three-dimensional turbulent boundary layer on the body for Re = 10^7 and 10^9. In both cases the flow parameters were calculated for an ordinary viscous fluid ($\alpha = 0$) and a weak polymer solution ($\alpha = 10$). The velocity of the body U_0 was taken equal to 1.15 m/sec ($v^*_0/U_0 = 0.02$) at Re = 10^7 and equal to 8 m/sec ($v^*_0/U_0 = 0.0029$) at Re = 10^9. These parameters approximately correspond to motion in a WSR-301 Polyox solution with a mass concentration $c = 10^{-5}$. The initial data for the calculations were determined by conversion of the experimental data [10] in proportion to the change in

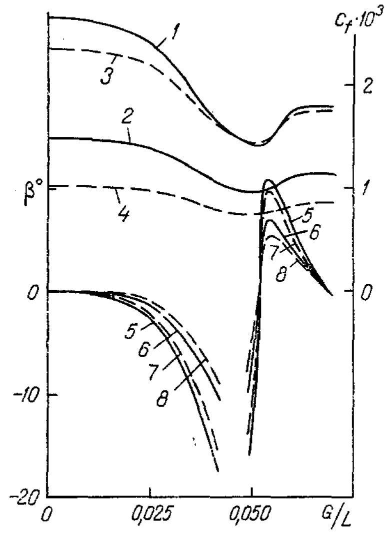

Fig. 4. Distribution of local coefficient of friction c_f (1-4) and downwash angle β (5-8) over perimeter of cross section at x_1 = 0.425; for remaining notation see Fig. 3.

the corresponding parameters of the turbulent boundary layer on a flat plate for the values of Re and α in question. The results of the calculations are reproduced in Figs. 3 and 4.

The data obtained indicates that when the Reynolds number is increased or polymer is added to the flow, the characteristic boundary layer thicknesses and the local coefficient of friction decrease. In the presence of considerable positive longitudinal pressure gradients zones in which the local coefficient of friction is greater in the polymer solution than for motion of the body in an ordinary fluid are observed in the three-dimensional turbulent boundary layer, as in the case of two-dimensional flows (curves 5 and 7 in Fig. 3). It should be noted that for fairly similar distributions of u_δ along the streamlines 1 and 5 the presence of three-dimensionality leads to a qualitatively different variation of the integral thicknesses and the local coefficient of friction along the body length. The c_f distribution over the perimeter of the cross section reveals a minimum in the zone of large negative values of the transverse curvature of the body. In the same region the downwash angle is observed to change sign. An increase in the Reynolds number or the introduction of polymer into the flow leads to a decrease in the absolute values of the downwash angles β. It is worth noting that the zone in which the c_f values in the polymer solution are greater than the corresponding values in an ordinary viscous fluid is localized and occupies only part of the perimeter of the body cross section.

NOTATION

X, Y, Z, longitudinal, transverse and surface-normal curvilinear coordinates; x, y, z, dimensionless curvilinear surface orthogonal coordinates; x_1, y_1, z_1, dimensionless Cartesian coordinates; L, body length; G, perimeter of the body cross section; K_{21} and K_{12}, geodesic curvatures of the equipotential lines and streamlines on the body surface; k_{21} and k_{12}, corresponding dimensionless values; $k_1 = u_\delta^{-1}$, h_2, h_3 = 1, Lamé parameters; U, longitudinal velocity component; V, normal velocity component; U_0, freestream velocity; u_δ, dimensionless velocity at the outer edge of the boundary layer (potential flow velocity); v^*, dynamic velocity; v^*_0, threshold dynamic velocity; τ_{wx} and τ_{wy}, longitudinal and transverse components of the wall shear stress; c_f and c_f^*, longitudinal and transverse local skin friction coefficients; δ, δ_1, δ_2, δ_{11}, δ_{12}, δ_{21}, and δ_{22}, dimensionless boundary layer thickness and displacement and momentum thicknesses; ρ, fluid density; ν, kinematic viscosity; Re, Reynolds number; E, mass entrainment function; H_1, shape factor; β, downwash angle; Π, Coles parameter; ϰ and B_0, constants of turbulence; and B, additive function in the logarithmic law.

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CONJUGATE PROBLEM OF EVAPORATION IN A LONG CHANNEL AT SMALL REYNOLDS NUMBERS

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The flow characteristics associated with evaporation from the walls of a long flat channel are investigated and the pressure and evaporation rate distribution along the channel are determined with allowance for the redistribution of energy in the walls.

In designing heat exchangers and drying chambers and in constructing models of evaporation from porous bodies it is necessary to consider problems of internal evaporation. A number of features of these problems are manifested in the simple case of evaporation from the walls of a long flat channel.

In the general case, this problem must be examined in the conjugate formulation, with allowance for heat transfer in the walls and the evaporation kinetics, as well as the vapor flow characteristics [1, 2]. In the case of high thermal conductivity of the solid phase and a short slot, for slow flows the fact that evaporation is nonequilibrium in character (allowance for difference between vapor pressure and saturation pressure) may have an important influence on the distribution of evaporation rate along the channel. At low heat fluxes for bodies with a high solid-phase thermal conductivity and low heat of evaporation it is also necessary to take into account the energy redistribution in the walls, which leads to nonuniform evaporation. The evaporation regimes were analyzed and the possible formulations of the problem classified in [2].

If the thermal conductivity of the solid phase is low, evaporation will be "uniform," i.e., the evaporation rate is determined starting from the heat flux supplied. The problem of the sublimation of ice was considered within the framework of this approximation in [3] for a molecular-viscosity vapor flow regime. A method that makes it possible to calculate the pressure distribution along a slot of finite width (with allowance for slip and temperature jump) was proposed and an experimental investigation was carried out. In making the calculations the pressure at the channel outlet was taken from the experimental results. It was shown that the pressure in the slot may be several times greater than the chamber pressure.

However, the initial equations of [3] were written for an incompressible fluid, although the pressure differences are such that the gas must be considered compressible. Evaporation was assumed to be uniform and equilibrium. At the same time, it is useful to analyze the case of nonuniform evaporation, when the energy redistribution in the walls is important, with allowance for the compressibility of the gas.

1. We will consider the conjugate problem of equilibrium evaporation from the walls of a long flat channel with a convergent nozzle at the outlet (width of slot 2h much less than

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